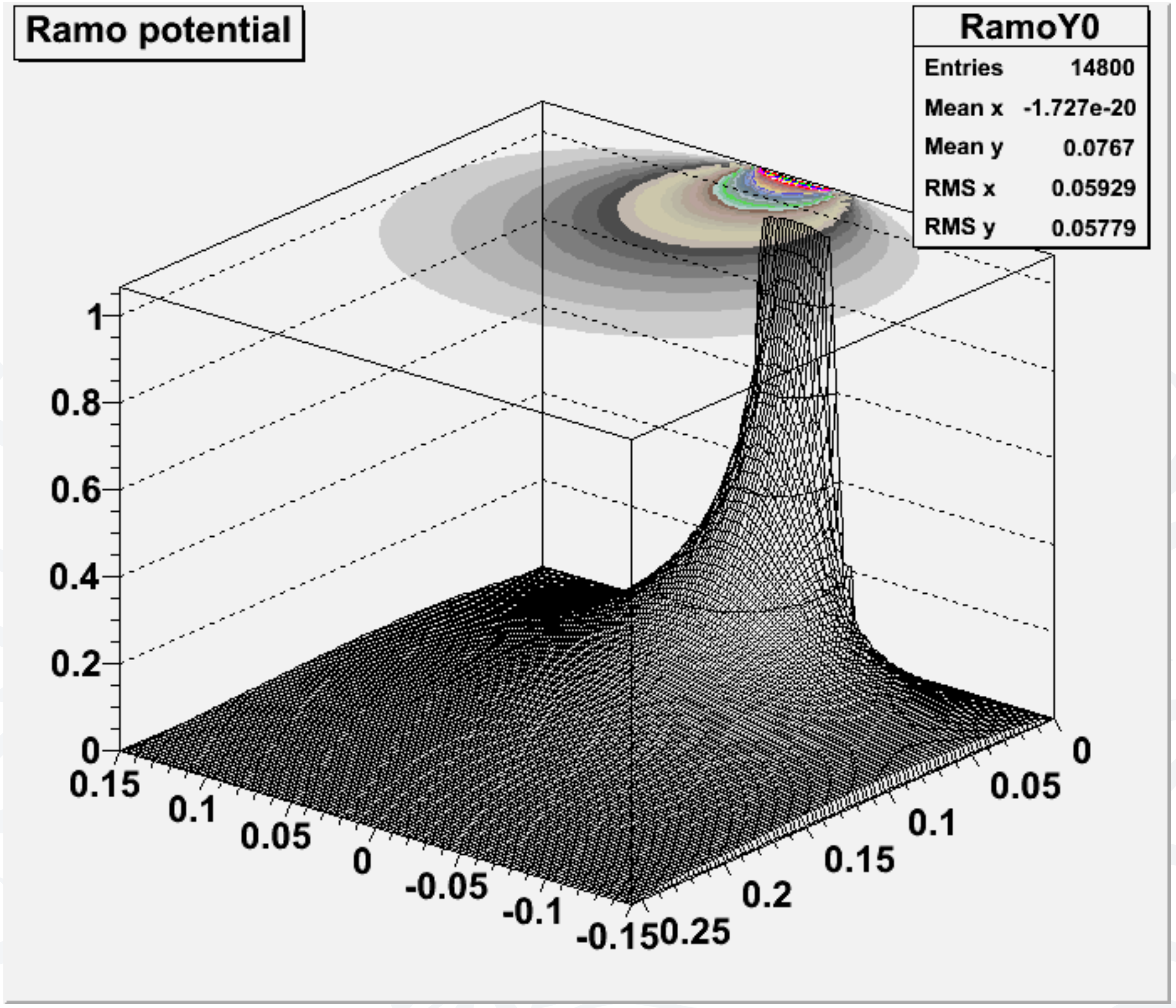
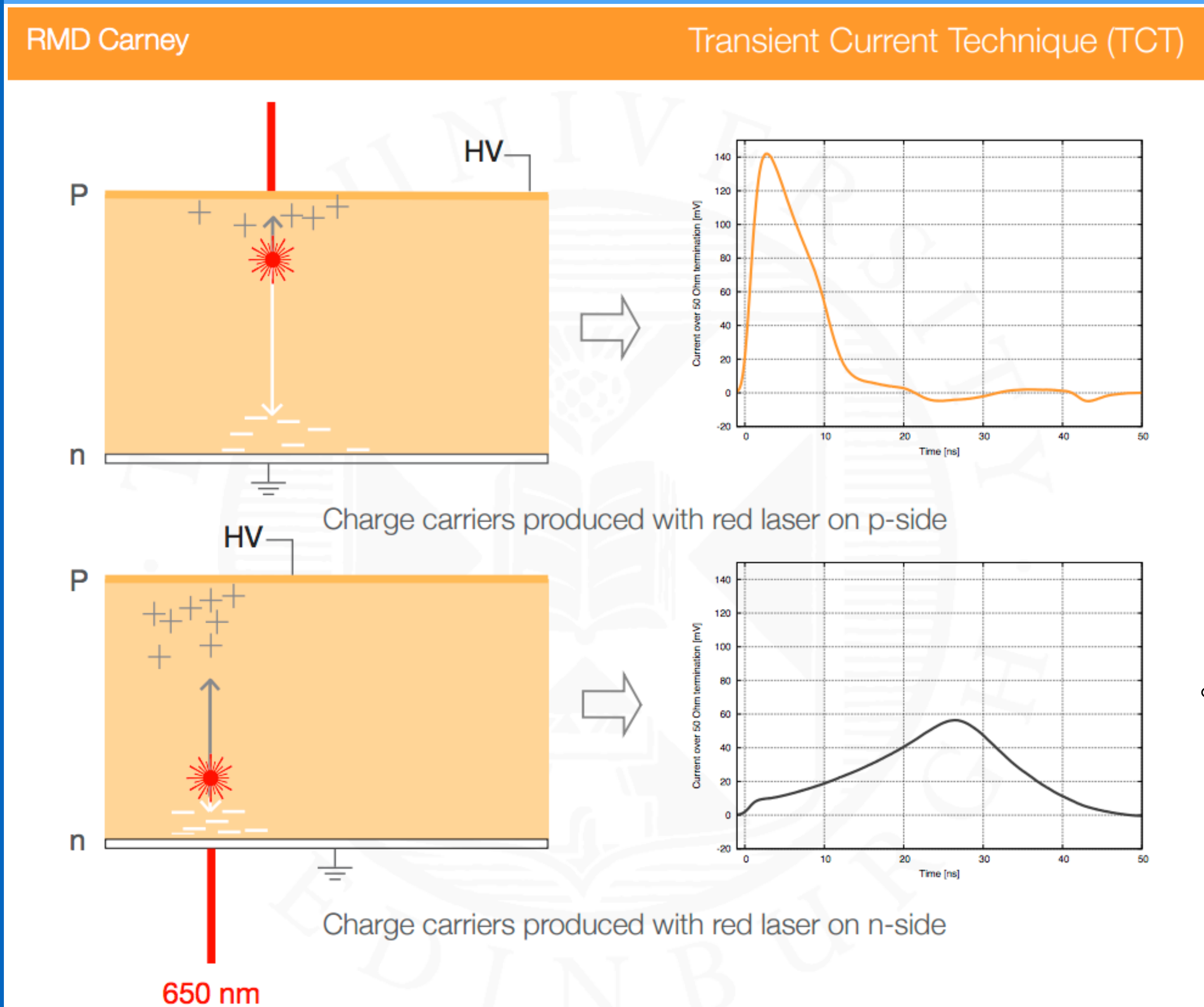
Ramo-Shockley Theorem



https://twiki.cern.ch/twiki/bin/viewauth/Atlas/PixelRadDamage

What is this picture of?
Why does it matter?
Why does it matter *particularly* for radiation damage studies?

Setting the stage: ionizing radiation in depleted semiconductor bulk



- Ionising interactions liberate free charge carriers in the depleted bulk of the material.
- The bias across the device creates an electric field which the charge carriers move within, holes in one direction, electrons in the other.
- This movement within the bulk induces a current on the collecting electrodes. This current continues until the free charge carriers recombine at their respective electrodes.

How does this relate to our simulation?

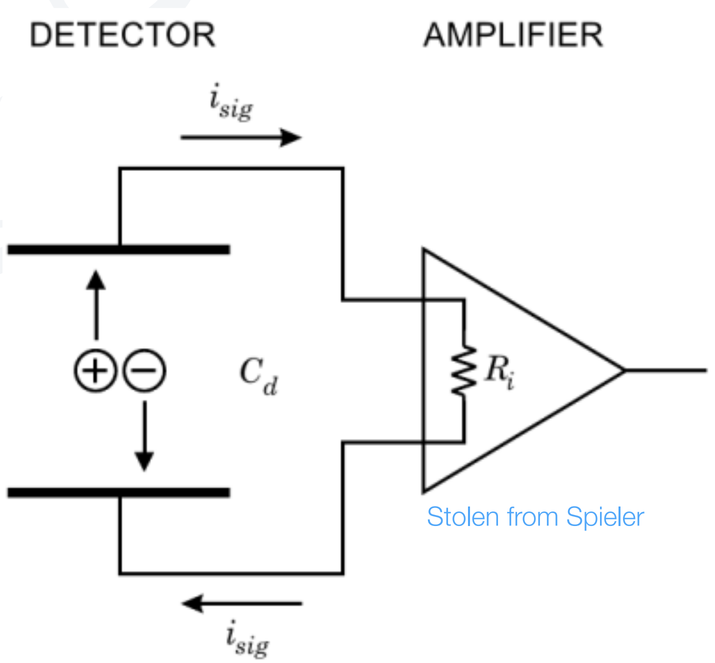
- Before the digitizer ‘digitizes’ it must create an analog signal in the detector based on the particle type, energy, and path length within the detector volume.

Misconceptions and charge collections

It sort of rhymes!

I think the problem comes from the term 'collected charge', which sounds as if the detector only produces a signal when the free charge carriers are 'collected' (recombine) at the electrode. But what would that assumption really affect (aside from being unphysical)?

- It would imply an offset in time from the generation of charge carriers (the particle interacting with the detector) and the signal generation. But the offset would only be of the order of 10 ns (0.4 bc) for electrons and 50 ns (2 bc) for holes. The shape of the signal would also be all wrong of course.



Stolen from Spieler

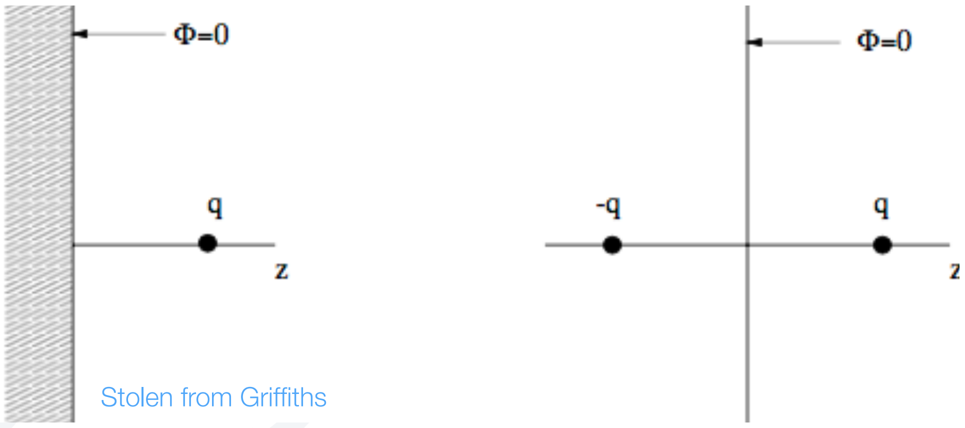
- But, then again the total 'collected' charge would be equivalent in both methods [3].

- However, if trapping is involved the two are no longer equivalent. Up to the point of being trapped the charge carrier does induce a current on the collecting electrode but this contribution is *lost* in the traditional model which only takes into account charge that is collected at the electrode.

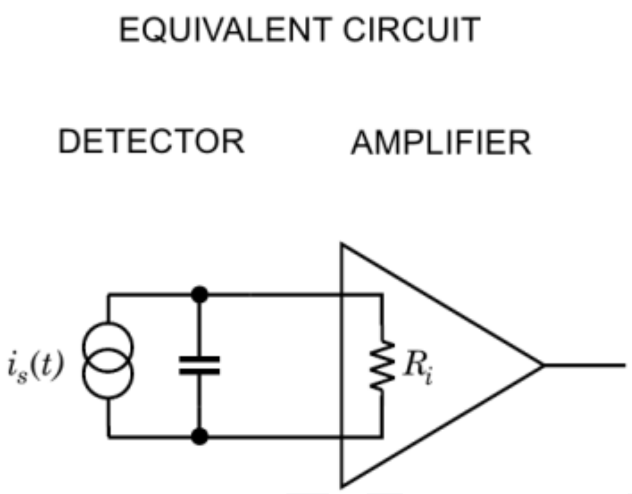
Induced charge

What affect does free charge have on electrodes?

- Using the method of image charges and Gauss's law we can quickly tell that the induced charge on a plate from a test charge, q, is -q.

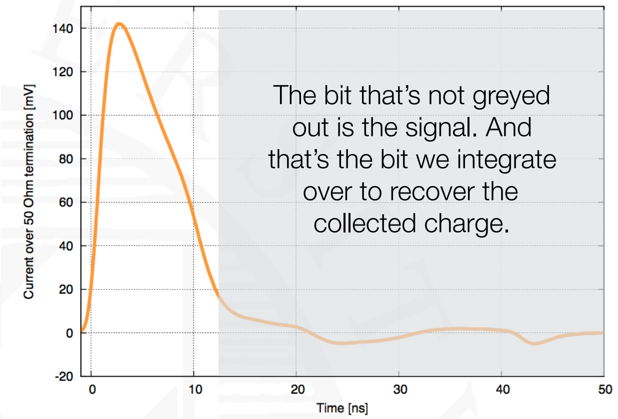


Stolen from Griffiths



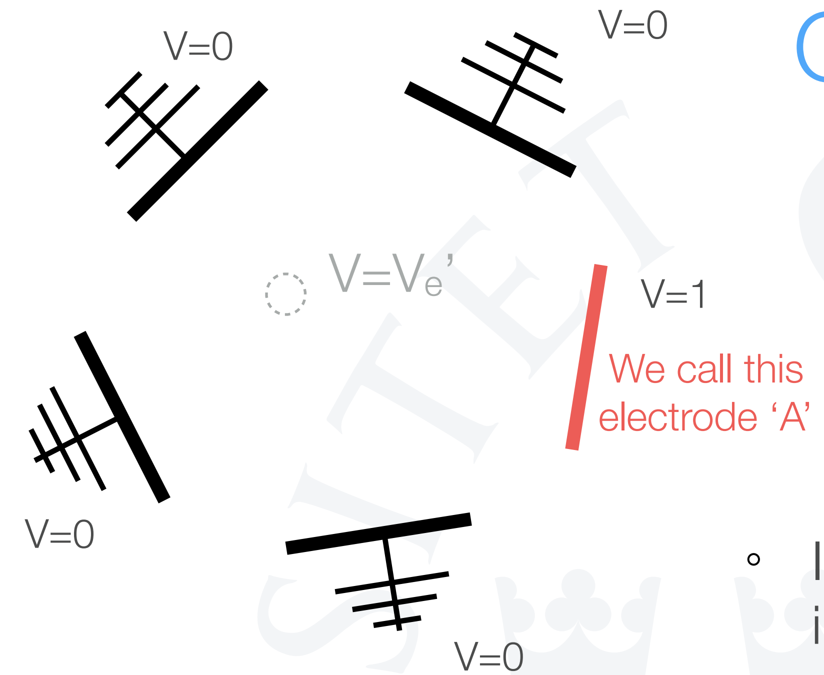
- We cannot observe this charge directly but we can measure its change of position. If, instead of one plate, the test charge is between two that are connected to form a closed circuit, the "change in induced charge manifests itself as a current". [2]

- And in integrating the induced current as the charge travels in the electric field we obtain the total charge (this is how we recover the charge using ToT and the threshold setting on the discriminator in post-processing).



- Prior to Ramo-Shockley in the 1930's it seems that calculating the induced current/charge was done by calculating the energy balance in moving a charge along the electric field lines and collocating the corresponding change in voltage across the detectors… (which has all sorts of problems). [2] Or by solving Gauss's Law for all the geometries at each point..
- To do this completely the total Electric field due to bias voltages, fixed, and moving charges at every time instance has to be calculated. Which is lengthy and totally impractical for complex geometries. [1]

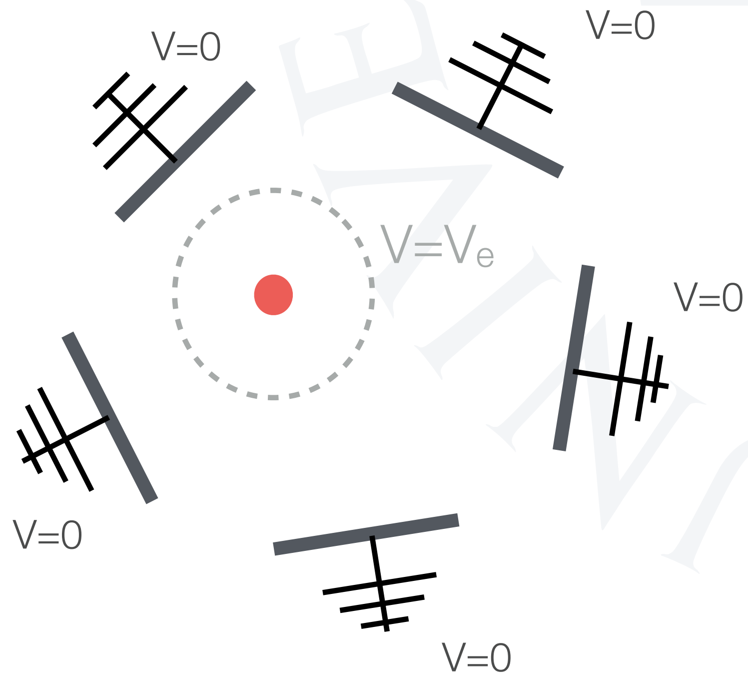
Ramo-Shockley derived



Calculate the Weighting Field

"[Weighting field] is the component in the direction of [velocity of charge carrier] of the electric field that would exist [if] the electron were removed, the electrode raised to unit potential, all other conductors grounded."[4]

- In this scenario there is no space charge at all. The only information about electric fields is derived from the geometry of electrodes which makes it much easier to calculate.



Charge carrier from Gauss' Law

- In this scenario all the electrodes are grounded and only the charge carrier remains. Gauss's law is used to calculate the flux of the carrier through a Gaussian surface (pictured here with potential V_e .

The divergence theorem

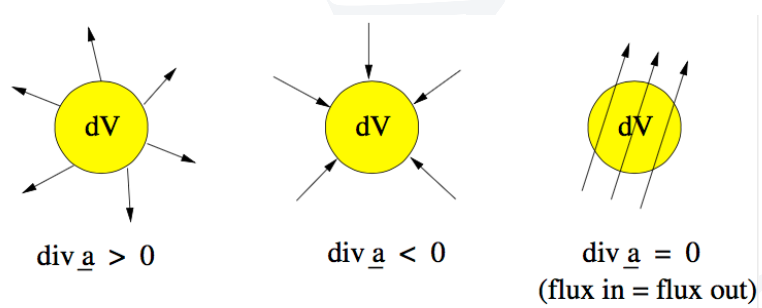
I totally forgot how Green's theorem worked so I went through its derivation again. Feel free to skip if you remember!
Formally, Green's Theorem relates the integral over a planar surface, 'A', to the line integral around the curve enclosing A.
But it's best understood as a rewriting of the divergence theorem (remember this?):

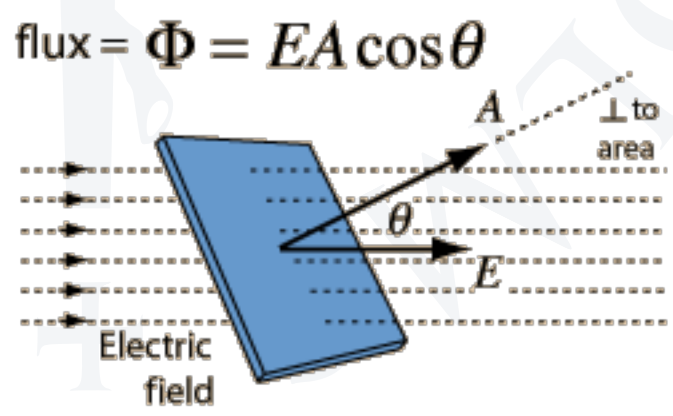
$$\int_V \underbrace{\vec{\nabla}\cdot\vec{a}} = \int_S \underbrace{\vec{a}\cdot d\vec{S}}$$

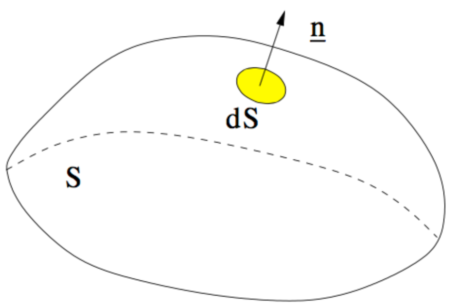
d**S** is the infinitesimal element of the surface, S, in the direction normal to S.

divergence of a vector field '**a**' within the volume *V*

The flux of **a** through the surface S.







Green's Theorem

After some manipulation (Ramo's original source is very explicit: https://archive.org/stream/2mathematicalthe00jeanuoft#page/160/mode/2up) we can get Green's theorem in this form:

$$\int_{volume} \underbrace{(V'\nabla^2 V - V\nabla^2 V')}\, dv = -\int_{surf} \underbrace{\left(V'\frac{\delta V}{\delta n} - V\frac{\delta V'}{\delta n}\right)} ds$$

Let the volume here be bounded by all conductors and the equipotential sphere around the charge. Then both terms with the laplacian go to zero as they contain no net charge.

This bit can be split up into 3 parts for the 3 geometries..

Perform integral over all surfaces except A.

Integral = 0, as V=V'=0 on these surfaces (we put them all to ground!).

+

Perform integral over electrode A.
Recall that V'=1 (the only electrode at unity) and V=0 (when there was a test charge present):

$$-\int_{surf\ A} \frac{\delta V}{\delta n} ds$$

+

Perform surface of equipotential sphere. The second term will be 0 since the test charge was removed:

$$-V_e' \int_{sphere} \frac{\delta V}{\delta n} ds$$

Adding the integrals

Adding the three integrals from the right hand side of Green's theorem we get the following:

$$0 = -\int_A \frac{\delta V}{\delta n} ds - V_e' \int_{sphere} \frac{\delta V}{\delta n} ds$$

This is defined as the 'weighting' or (Ramo) potential

We define the charge on electrode A as the induced charge, Q_A

$$= 4\pi Q_A + 4\pi e V_e'$$

The gaussian surface was defined to enclose the test charge, e

$$\boxed{\therefore Q_A = -eV_e'}$$

The induced current is then just the change in induced charge over time:

$$i_A = \frac{dQ_A}{dt} = -e\frac{dV_e'}{dt} = -e\left[\frac{\delta V_e'}{\delta x}\frac{dx}{dt}\right]$$

This is defined as the 'weighting' or (Ramo) field. Note that is is NOT the electric field!!

Defining the velocity of the charge carrier as:

$$v = \frac{dx}{dt}$$

Then the induced current is:

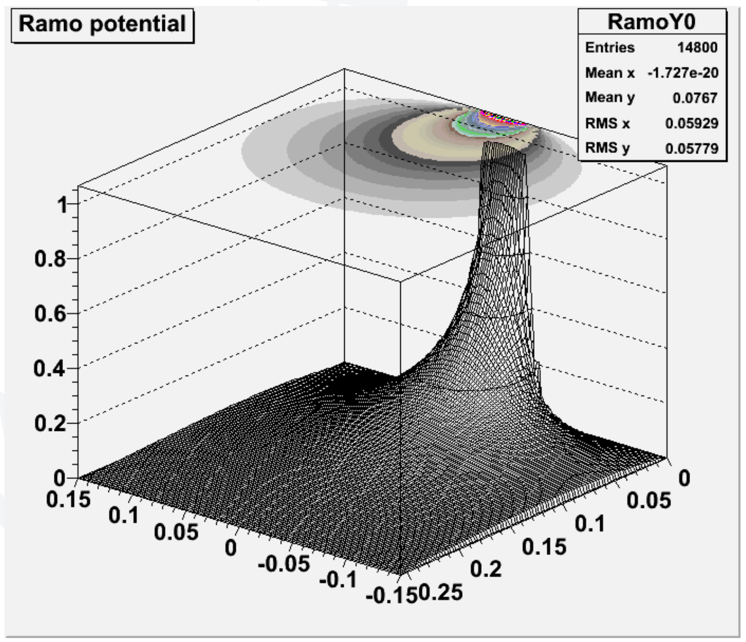
$$\boxed{\therefore i_A = e\vec{v} \cdot \vec{E}_v}$$

First, some key points

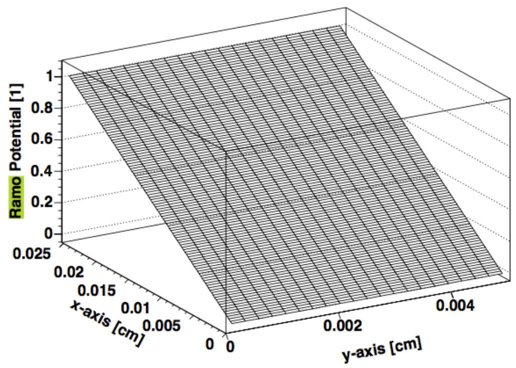
1. The Electric field determines the charge trajectory and velocity. Neither of these have anything to do with the weighting field, so if you want to know them at a particular point you must go through the entire process using Gauss's law, etc.
2. The weighting field depends only on the geometry of the electrodes and determines how the charge's motion couples to a specific electrode to induce a signal.
3. Only in 2-electrode configurations are the weighting field and Electric field the same shape (and they are still only proportional not identical)![2]

The weighting field for smaller electrodes is weird

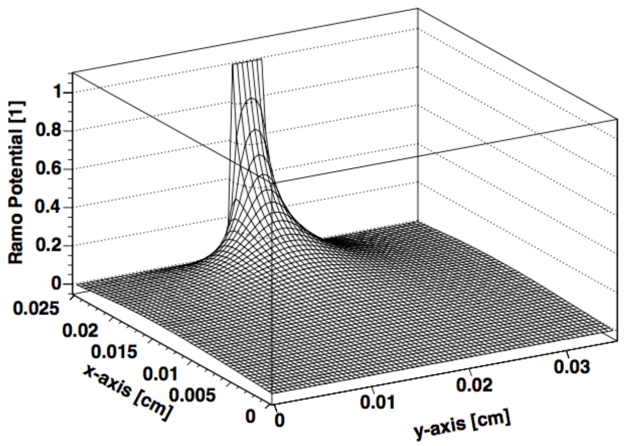
So, that's what this picture is! It's the weighting field for a pixel.
It's interesting because it shows that the majority of contribution to the signal is localized directly under the pixel.



Weighting potential: A single pixel



Weighting potential: 2 ∞-planar electrodes



Weighting potential: ∞ strip

But.. what does it mean?

And how this links back to radiation damage!

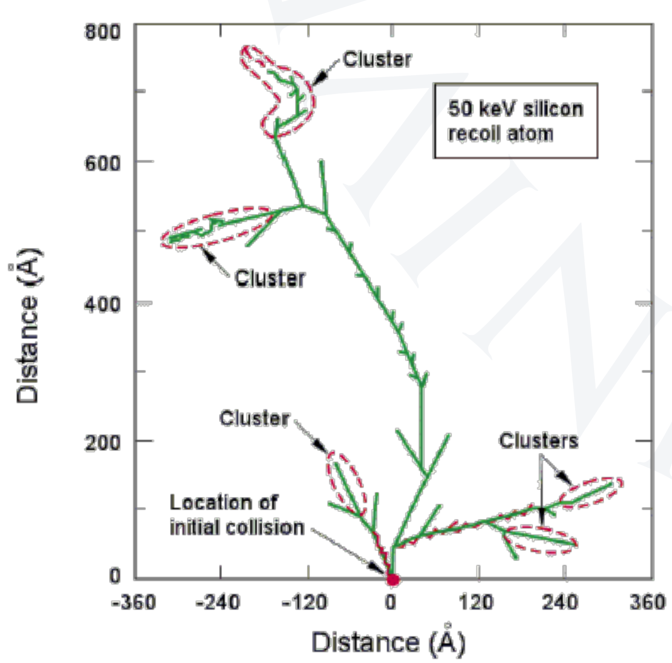
Faster calculation, more relevant information

In the current digitizer (in Athena) we get to the induced charge in a roundabout way. We drift and diffuse the charge carriers until they hit the electrodes. It's lengthy and numerical.

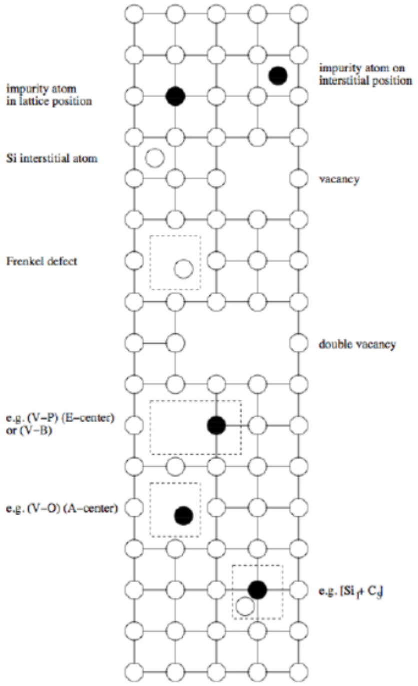
With this method we can pre-calculate the weighting field just using our knowledge of the geometry of the electrodes! So at run time it's a simple lookup to see if the charge carrier is contributing to the signal!

Radiation damage

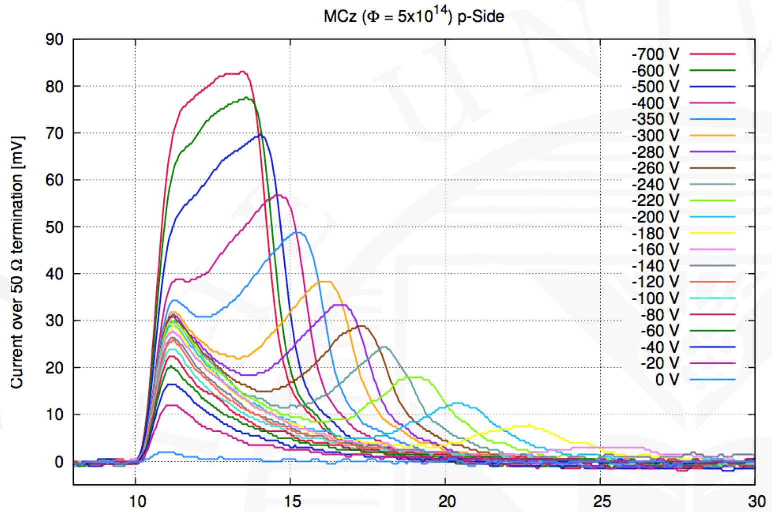
Which is important, because charge carriers might contribute to the induced signal before being trapped. If we didn't do this we'd be simulating less signal in the detector than an equivalent event would actually generate.



NIEL damage: path of destruction



NIEL damage: types of defects



The shape of the electric field after radiation damage as mapped out by electrons

References

- [1] W. Shockley, *Currents to Conductors Induced by a Moving Point Charge* [http://dx.doi.org/10.1063/1.1710367].
- [2] Helmuth Spieler, *Semiconductor Detector Systems:* Section 2.5 (pg. 71-83) [Google books]
- [3] Glenn Knoll, *Radiation Detection and Measurement:* 3rd edition Appendix D.
- [4] Simon Ramo, *Currents Induced by Electron Motion* [10.1109/JRPROC.1939.228757]
- [5] Olaf Krasel, *Charge Collection in Irradiated Silicon-Detectors* [http://d-nb.info/1011531879/34]